

The Explanatory Indispensability Argument

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Abstract:

A new version of the indispensability argument in the philosophy of mathematics, relying in part on recent work in mathematical explanation, alleges that our mathematical beliefs are justified by their indispensable appearances in scientific explanations. I first distinguish the explanatory argument from the traditional, Quinean indispensability argument, characterizing the sense of ‘explanation’ on which the new argument depends. I then argue that, given this sense of ‘explanation’, the premise of the new argument that has received most attention, the claim that there are mathematical explanations of physical phenomena, is actually uncontroversial. In contrast, the argument is weak at a premise which has not been sufficiently defended, the claim that our ontological commitments are found in our explanations. I conclude that the new explanatory indispensability argument is no improvement on the Quinean one.

The Explanatory Indispensability Argument

The thirty-year debate over the indispensability argument, following the appearance of Hartry Field's seminal *Science without Numbers*, appears to be stuck in a deadlock. Field's attempt to rewrite Newtonian Gravitational Theory quantifying over space-time regions rather than real numbers, and John Burgess's later improvements, gave hope to dispensabilists.¹ Further advances, like Mark Balaguer's sketch of a dispensabilist project for quantum mechanics, supported the cause. Additionally, Burgess and Rosen's argument that the lack of dispensabilist projects currently available is weak evidence for their eventual non-existence provided the dispensabilist some solace in the face of difficulties.² While it is pretty clear that no neat, first-order theory which eschews all mathematical axioms will suffice for all of current and future science, the dispensabilist has reasonable hope of finding moderately attractive reformulations of large swaths of scientific theory.

Indispensabilists like Mark Colyvan have been both emboldened by the lack of convincing success on the side of the dispensabilists, and eager to fortify the original argument. According to a new explanatory indispensability argument, we should believe in mathematical objects because of their indispensable roles in our scientific explanations. Alan Baker defends the explanatory argument, and we can see versions of it in Colyvan's work.³ Paolo Mancosu states the argument explicitly:

¹ Some terminology, as I am using it: a *platonist* believes in the existence of mathematical objects. An *indispensabilist* is a platonist who justifies his/her belief in mathematical objects by the ineliminable appearance of mathematical objects (in some discourse or other to be specified, e.g. scientific theory, metalogic, mathematical explanation). A *dispensabilist project* is a reformulation of some discourse which eliminates references to mathematical objects. A *dispensabilist* is someone who would believe in the existence of mathematical objects, if they were indispensable (from some discourse or other), but who believes that there can be successful dispensabilist projects (in that discourse). A *nominalist* denies the existence of mathematical objects.

² See Burgess and Rosen 1997: 118.

³ See Baker 2001: 211; Baker 2005: 224-5; Lyon and Colyvan 2008: 242; and Colyvan 2007: 119-122. Colyvan's work straddles the line between an explanatory indispensability argument, and a traditional indispensability argument which takes explanatory strength as one among several theoretical virtues.

- EI EI1. There are genuinely mathematical explanations of empirical phenomena.
- EI2. We ought to be committed to the theoretical posits postulated by such explanations.
- EIC. We ought to be committed to the entities postulated by the mathematics in question (Mancosu 2008: §3.2).

EI differs from the traditional indispensability argument by focusing on the role of mathematics in scientific explanations, rather than in scientific theories. Indeed, the original Quinean indispensability argument relies on the claim, distinct from EI2, that we find our ontological commitments in our best theories.

- QI QI1. We should believe the theory which best accounts for our sense experience.
- QI2. If we believe a theory, we must believe in its ontological commitments.
- QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
- QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
- QIC. We should believe that mathematical objects exist.⁴

The differences between QI and EI may seem, at first, of little importance. If we take the instances of ‘explanations’ in EI to refer to scientific explanations in a standard sense, there is no significant difference between a theory and an explanation. For example, a D-N explanation of a phenomenon P proceeds by proffering the laws of a serious theory, combined with appropriate initial conditions, from which a description of P is derived according to standard deductive rules. The theories to which such explanations appeal are ones in which we speak most strictly, ones in which we take care to refer only to our most sincere commitments. Modifications of this traditional model, like Railton’s model of probabilistic explanation, or Kitcher’s unificationist account, work similarly. Kitcher, for example, relies on unifying argument patterns which also answer why-questions by proffering inferences made within a serious theory. Taking explanatory power as one of several theoretical virtues, like simplicity and parsimony, that scientists seek to optimize, Field and others focused on whether our best

⁴ See Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986.

theories can be reformulated to avoid mathematical references. This question has led to the current stalemate.

In contrast, proponents of EI think they have found a way to break the deadlock. Even if the nominalist can reformulate scientific theories to avoid mathematical commitments, they argue, the new theories lack the explanatory power of the originals. “Even if nominalisation via [a dispensabilist construction] is possible, the resulting theory is likely to be less explanatory; there is explanatory power in phase-space formulations of theories, and this explanatory power does not seem recoverable in alternative formulations” (Lyon and Colyvan 2008: 242).

Such claims in favor of EI are non-starters, if the sense of ‘explanation’ used in EI is traditional. First, for reasons I will not discuss, standard accounts of scientific explanation do not comfortably apply to mathematical explanation.⁵ Second, EI must rely on a different notion of explanation if we are to take it as distinct from QI. It is a standard requirement of any dispensabilist project that the mathematized theory be able to derive no further conclusions than the nominalist reformulation.⁶ That is the point of Field’s attempts to construct representation theorems, and his more general arguments for the conservativeness of mathematics. One just could not have a successful nominalization of a scientific theory with less explanatory power, unless one is using a different sense of ‘explanation’.

An alternative sense of ‘explanation’ that could ground EI involves subjective understanding. Unlike standard scientific theories, dispensabilist reformulations will be imperspicuous, not useful to working scientists. The awkwardness of dispensabilist reformulations is granted by Field and other

⁵ Briefly, mathematics contains too many entailments of the requisite type, some of which are clearly not explanatory. Some mathematicians and philosophers distinguish between explanatory and non-explanatory proofs. See Mancosu 2001: §1.1 and Mancosu 2008: §5-6.

⁶ Or, anyway, no further *important* conclusions. See the exchange between Maddy 1990 and Field 1990, rooted in criticisms found in Shapiro 1983 and rumored to have originated in unpublished work by Saul Kripke. More recently, see Melia 2000 for concerns about the in-principle incompleteness of dispensabilist reformulations.

dispensabilists who generally do not suggest that scientists adopt the reformulations. The dispensabilist grants that standard theories are more comprehensible, more explanatory in this alternative sense. So, it is reasonable to expect that dispensabilist reformulations will lose explanatory power, in this sense, while retaining all (or most) of the inferential relations of their corresponding standard theories.

I will thus take the sense of ‘explanation’ in which dispensabilist projects, even if successful responses to QI, are not just as explanatory as their standard counterparts to involve subjective understanding, rather than deductive strength. I will call this an epistemic sense of ‘explanation’.

The distinction between taking explanation to be a theoretical virtue, and thus working with a traditional Quinean argument, and looking at the indispensability of mathematics for epistemic explanations is subtle, but important. The availability of a dispensabilist reformulation of a standard scientific theory is essential to evaluating QI, but it is completely irrelevant to whether EI succeeds. A dispensabilist reformulation of a standard scientific theory which preserves deductive strength shows QI⁴ to be false; QI fails. In contrast, such reformulations do not show that we can eliminate mathematical objects from our mathematical explanations (if there are any) of physical phenomena, in the epistemic sense of explanation; EI remains open.⁷

The debate over EI has focused on EI1, on whether there are genuinely mathematical explanations of physical phenomenon. My first goal in this paper was to clarify the sense of ‘explanation’ being used in EI. Having done so, I will argue that the lively discussions of EI1 are mainly

⁷ Two qualifications. First, deductive strength is not the only criterion for success of a dispensabilist project. Still, (epistemic) explanatory merit has played an insignificant role in the debate over QI, since the central question has been whether one can eliminate quantification over numbers within a reasonable logical framework. Second, I present EI as an option for the platonist, and thus a demand on the nominalist, who must show how we can eliminate mathematics from both theories and explanations. Alternatively, we could see it as an additional demand on the platonist: even if dispensabilist constructions do not work, if there are no genuinely mathematical explanations we should withhold commitments to mathematical objects. See Bangu 2008, and Melia 1998: 70. Melia 2002 and Leng 2005: 179, though working with explanation as a theoretical virtue, can also be seen as taking this latter route. My criticisms of EI are neutral between the two views.

irrelevant to the success of EI.

A wide range of lively examples of purported mathematical explanations of physical phenomena have been discussed, many presented by Mark Colyvan.⁸ The Borsuk-Ulam topological theorem explains the existence of two pressure/temperature antipodes in the Earth's atmosphere; that π is transcendental explains why we can not square the circle; Simpson's paradox may help explain the persistence of maladaptive traits like altruism. At face value, such examples provide compelling, if unsurprising, evidence for EI1, especially when 'explanation' is taken in the epistemic sense. In the standard sense, explanations are tied to our most austere, parsimonious theories. We are required to ask whether it is possible to re-describe some of the phenomena or explanations to eliminate their mathematical elements. In contrast, when our goal is an epistemic explanation, we are free to invoke fiction and metaphor. The question of whether such examples can be reformulated to eliminate mathematical objects is precisely what EI is designed to avoid.

Alan Baker's cicada example may be the most influential case used to support EI1. Three species of cicadas of the genus *Magicicada* share a life cycle of either thirteen or seventeen years, depending on the environment. Baker claims that the phenomenon of having prime-numbered life-cycles may be explained thus:

- CP CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.
- CP2. Prime periods minimize intersection.
- CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.
- CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
- CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods (Baker 2005: 233).

Baker argues that the mathematical explanans, at CP2, supports the "mixed' biological/

⁸ See Colyvan 2001: 81-6 and Colyvan 2007: 120-1.

mathematical law” at CP3, which explains the empirical claim CP5. Sorin Bangu argues that the explanandum in question at CP5 is, like CP3, composed of both mathematical and physical facts: a physical phenomenon (the time interval between successive occurrences of cicadas); the concept of a life-cycle period; the number seventeen; and the mathematical property of primeness. The mathematical facts only explain the mathematical portions of the explanandum. “If the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers” (Bangu 2008: 180). Further, it is question-begging to profess ontological commitments to mathematical objects on the basis of their use in mathematics.

Bangu correctly argues that an explanandum with a mathematical element weakens the claim that CP supports EI1. But, his charge of circularity is too strong. CP is, as it stands, an explanation of a biological fact which refers to mathematical objects. Bangu’s allegation that the mathematical elements of CP1-4 only explain the mathematical portion of CP5 depends on whether he can analyze, or reformulate, CP5 to separate the mathematical portion from the empirical remainder. If the mathematical elements of CP5 were inseparable, then we could conclude, with the indispensabilist, that there are essentially mathematical elements of our descriptions of physical phenomena. The mathematical explanations of those elements will thus contribute ineliminably, or indispensably, to our explanations of the phenomenon. If the mathematical elements of the explanandum are ineliminable, then we have reason to believe that the world is essentially as the indispensabilist alleges. In other words, Bangu’s claim that the mathematical elements of CP1-4 explain only the mathematical portion of the explanandum begs the question against the indispensabilist of whether the mathematical portion is essential to a description of the phenomenon.

Conversely, if we can, in a dispensabilist spirit, eliminate the mathematical elements of CP, then we can, with Bangu, deny that it supports EI. In fact, the elementary uses of numbers in CP are easily

excisable. It is only drudgery to remove the adjectival uses of whole numbers in CP4 and CP5. The concept of primeness in CP2 and CP3 requires a bit more machinery, but, as Mary Leng observes, it does not even demand a completed ω -sequence.⁹

Bangu's criticism thus recalls the dialectic between the indispensabilist and the dispensabilist. If we can construct explanations of physical phenomena which eliminate references to mathematical elements, then there are no essentially mathematical explanations of physical phenomena, and EI1 fails. If we can not reformulate our explanations to eliminate references to mathematical objects, then we have support for EI1. But the whole point of introducing EI was to avoid precisely this dispute.

Bangu's argument does not undermine the claim that there are mathematical explanations of physical phenomena, in the sense required by the proponent of EI. By recasting our explanations to remove references to mathematical objects, we trade a satisfying (epistemic) explanation for an austere, parsimonious theory. And the theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world.

Herein lies the real weakness of EI. EI2 states that we ought to be committed to the theoretical posits postulated by mathematical explanations of physical phenomena. Once we realize that the sense of 'explanation' in question is epistemic, any force that EI2 is supposed to have is lost. Consider whether the following inference should convince someone us there are numbers.

IM I have two mangoes.
 Andrés has three different mangoes.
 So, together we have five mangoes.

⁹ See Leng 2005: 186.

The answer is clearly negative, since simple adjectival uses of arithmetic are easily eliminated.

$$\begin{aligned}
 \text{IN} \quad & (\exists x)(\exists y)(Mx \cdot My \cdot Bxm \cdot Bym \cdot x \neq y) \\
 & (\exists x)(\exists y)(\exists z)(Mx \cdot My \cdot Mz \cdot Bxa \cdot Bya \cdot Bza \cdot x \neq y \cdot x \neq z \cdot y \neq z) \\
 & (x)[(Mx \cdot Bxa) \supset \sim Bxm] \\
 & \therefore (\exists x)(\exists y)(\exists z)(\exists w)(\exists v)(Mx \cdot My \cdot Mz \cdot Mw \cdot Mv \cdot x \neq y \cdot x \neq z \cdot x \neq w \cdot x \neq v \cdot y \neq z \cdot \\
 & \quad y \neq w \cdot y \neq v \cdot z \neq w \cdot z \neq v \cdot w \neq v)
 \end{aligned}$$

If someone were to present IM in defense of platonism, we would justifiably respond with IN.

Whether we believe in mathematical objects or not, IM is not a good reason for believing in them, since it is just loose talk which does not reflect our serious commitments. If we want to display our actual ontological commitments, we must speak soberly, invoke parsimony, and rewrite our casual sentences. We construct inferences like IN which make it clear that, strictly speaking, the subjects of the given inference are mangoes, not numbers.

Now, consider the question, “Why are there five mangoes here?” A sufficient explanation, in the epistemic sense, is that I brought two and Andrés brought three. That fact is explained by IM, and only awkwardly demonstrated, if explained at all, by IN. IM is not a complete, best theory of these mangoes, of course. It requires background assumptions about object constancy, and that mangoes do not annihilate each other when, say, there are more than three together. But, it will satisfy any ordinary person, more so than IN. In fact, the only way for IN to have any plausible epistemic explanatory force is for it to be translated back to something like IM.

IM and IN exemplify the two distinct senses of ‘explanation’ I have discussed. IM is a genuinely mathematical explanation, in the epistemic sense, of an empirical phenomenon, but we are not compelled to take its mathematical references seriously. IN is preferable for the purposes of revealing ontological commitments, and is the kind of inference that could be used in a traditional, D-N or related, sense of ‘explanation’, but it contains no mathematical references. The contrast between IM and IN exemplifies, if overly simply, the contrast between our most epistemically explanatory statements and our most

austere theories. The physicist may appeal to the Bohr model of the atom in order to explain quantum energy states to a novice, but will quickly drop all references to the planetary model when speaking seriously.

There are two morals emphasized by the differences between IM and IN.

Moral 1: We are committed to mathematical objects not by our casual uses of numbers, but only when we are speaking most seriously.

Moral 2: The theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world, in the epistemic sense of 'explain'.

Taken together, these morals challenge EI at its second premise. To differentiate itself from the traditional indispensability argument, EI may rely on an epistemic sense of explanation. But there is little reason to believe that explanations which facilitate our subjective understanding reveal our ontological commitments. Quine's original indispensability argument received essential support from his claim that we find our ontological commitments precisely in our best theories. EI receives no such support, and is thus no improvement on the original argument. The old debate over QI, on which I have taken no position in this paper, remains salient.

My argument against EI assumes that it is based on an epistemic notion of explanation. I contrasted the epistemic sense of 'explanation' with a metaphysical sense of the term on which QI is based. Perhaps there is an independent sense of 'explanation' on which EI might be based, and on which it would be more successful.

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